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SCHEME DEPENDENCE AT SMALL x

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Abstract

We discuss the evolution of F_2^p at small x, emphasizing the uncertainties related to expansion, fitting, renormalization and factorization scheme dependence. We find that perturbative extrapolation from the measured region down to smaller x and lower Q^2 may become strongly scheme dependent.

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It is now well established[1] that the behaviour of F_2 in the region of small x and large Q^2 accessed by the HERA experiments[2] provides a confirmation of the double scaling behaviour[3] predicted asymptotically in perturbative QCD[4]. As the experimental accuracy improves, it is now possible to test the theory beyond this simple leading order prediction, by comparing the data to a full next-to-leading order (NLO) determination of the x and Q^2 dependence of F_2. This, however, requires a study of the renormalization and factorization scheme dependence which characterizes perturbative computations, and which at small x become particularly significant, due to the growth of anomalous dimensions and coefficient functions. Moreover, the presence in the problem of two large scales (Q^2 and $s = Q^2(1-x)/x$) requires the choice of an expansion scheme which sums up all the appropriate leading (and subleading) logarithms. Here we assess the size of these ambiguities and in particular discuss how they affect the computation of F_2 and, conversely, the extraction from F_2 of information on the form of parton distributions at small x.

Determining the evolution of structure functions by solution of the renormalization group equations in leading order corresponds to summing all logs of the form $\alpha_s^p(\log Q^2)^q(\log\frac{1}{x})^r$ with $p = q$ and $0 \leq r \leq p$; double scaling is a consequence of the dominance at small x of the contributions with $r = p = q$, i.e., such that the two large logs are treated symmetrically. It is in fact possible[5] to reorganize the perturbative expansion in such a way that the full LO contribution to anomalous dimensions treats the two logs symmetrically, i.e. such that in LO each power of α_s is accompanied by either of the two logs (that is, such that $1 \leq q \leq p$, $0 \leq r \leq p$, $1 \leq p \leq q+r$). This expansion scheme (the double leading scheme) can then be extended to NLO and beyond. Within any given scheme the structure functions are expressed as power series in α_s, even though solving the renormalization group equations sums contributions involving large logarithms to all orders in α_s.

Consistent solution of the evolution equations in any specified expansion scheme and to a given order is then (at least in principle) always possible. In practice, the anomalous dimensions are known through their Laurent expansion in N. All the LO coefficients of this expansion are known for the 2×2 matrix of singlet anomalous dimensions[6], but the NLO coefficients of the singular terms in N are only known for γ_N^{qg} and γ_N^{qq}[7]; the corresponding coefficients in γ_N^{gg} and γ_N^{gq} can however be fixed by requiring momentum conservation[8]. We will henceforth consider NLO computations in

the double-leading scheme.* More specifically perturbative evolution is performed in the usual loop expansion scheme down to a certain x_0, and then the double-leading scheme used below it. The value of the parameter x_0 can only be determined by comparison to experiment. Here we will consider two extreme double-leading NLO scenarios, namely x_0 smaller than any value of x covered by the HERA data, (i.e., in practice, $x_0 = 0$ or ordinary two-loop evolution), and $x_0 = 0.1$.

Once x_0 is fixed, renormalization and factorization schemes still have to be specified in order to perform NLO computations. Without loss of generality the renormalization scheme will be chosen to be $\overline{\mathrm{MS}}$: other renormalization schemes then correspond simply to a change of renormalization scale. The choice of factorization scheme is more complex. Firstly, we have a choice between schemes in which to all orders F_2 is directly proportional to the quark distribution (parton schemes, such as the DIS scheme), and schemes where (starting at NLO) F_2 receives a gluon contribution (such as the $\overline{\mathrm{MS}}$ scheme). Furthermore, in parton schemes the coefficients of NLO singular contributions to the singlet anomalous dimensions also depend on the choice of factorization scheme: when computed in the DIS scheme[7] they contain a process-independent singularity in the quark sector, which is removed if off-shell factorization is used instead ($\mathrm{Q_0}$DIS scheme)[10]. It is even possible to set the NLO singularities in the quark sector to zero, thereby factorizing the entire singularity into the starting distribution (SDIS scheme)[11].† Corresponding $\overline{\mathrm{MS}}$ schemes may be constructed by insisting that the anomalous dimensions be the same as those in the standard $\overline{\mathrm{MS}}$ scheme [7], but with the coefficients being adjusted accordingly (so that in particular in $\mathrm{Q_0}\overline{\mathrm{MS}}$ scheme the process - independent singularity is removed from the coefficient function).

Finally, there is still an ambiguity in the definition of the initial parton distributions (a 'fitting scheme' ambiguity), related to the fact that these can be fitted in a parton scheme or in an $\overline{\mathrm{MS}}$ scheme regardless of which scheme is chosen to evolve. Besides providing information on the dependence of

*Notice that this is not quite the same as the approach of ref. [9], where the higher order singularities are simply added to the one and two loop anomalous dimensions: in the double-leading expansion all the NLO terms may be treated consistently, by linearizing them in order to avoid spurious sub-subleading terms.

†It turns out that in the HERA region the results obtained in the SDIS scheme are essentially identical to those found by ordinary two loop evolution[8].

	norm.(%)		λ_q	λ_g	χ^2
a)	96	103	-0.23 ± 0.05	0.10 ± 0.07	57.3*
	97	103	-0.24 ± 0.05	0.12 ± 0.08	57.6
	94	101	-0.24 ± 0.09	-0.52 ± 0.23	59.2
	95	101	-0.25 ± 0.10	-0.49 ± 0.26	59.0*
b)	96	102	-0.25 ± 0.02	0.03 ± 0.16	64.5*
	97	104	-0.25 ± 0.02	-0.08 ± 0.01	58.1
	97	104	-0.12 ± 0.02	-0.01 ± 0.20	62.5
	97	103	-0.13 ± 0.07	-0.36 ± 0.24	57.9*
c)	96	102	-0.26 ± 0.02	0.12 ± 0.17	72.6*
	98	106	-0.24 ± 0.02	-0.17 ± 0.09	65.0
	97	103	0.10 ± 0.06	0.01 ± 0.37	73.1
	95	101	-0.03 ± 0.03	-0.75 ± 0.05	62.3*
d)	100	106	-0.13 ± 0.04	0.18 ± 0.01	63.4
	94	100	-0.22 ± 0.06	-0.10 ± 0.10	57.9
	100	107	-0.16 ± 0.11	0.07 ± 0.25	63.9
	89	95	-0.28 ± 0.05	-0.70 ± 0.12	73.9

Table 1: Fitted parameters for: a) $x_0 = 0$; b) $x_0 = 0.1$, Q_0 factorization; c) $x_0 = 0.1$, standard factorization. In each case the four entries correspond respectively to DIS distributions (DIS and $\overline{\rm MS}$ evolution); $\overline{\rm MS}$ distributions (DIS and $\overline{\rm MS}$ evolution). The two entries d) show the effect on the first and fourth entry of the table of varying the renormalization scale by a factor of two either side.

the results for F_2 on the specific choice of parton parametrization, varying the fitting scheme demonstrates the implicit scheme dependence of the fitted parameters.

The results of fitting F_2 to HERA data [2] are summarized in the table and displayed in the figure.[‡] The free parameters are the normalizations of the two data sets and the small-x exponents of the quark and gluon distributions,

[‡]The corresponding results of ref. [8] are determined by using a slightly different treatment of thresholds: here continuity of F_2 is imposed (continuity of DIS distributions) whereas there continuity of the $\overline{\rm MS}$ parton distributions was required instead. The slight variation of the results gives a feeling for the corresponding uncertainty.

which behave as x^λ as $x \to 0$; the resulting χ^2 (for 120 d.f.) is also given. All fits are performed with $\alpha_s(M_z) = 0.120$[8]; initial parton distributions are given at 2 GeV for the $x_0 = 0$ fits and 3 GeV for $x_0 = 0.1$.[§]

The results can be summarized as follows:

a) Whereas the inclusion of two loop corrections improves significantly the agreement of F_2 with the data, going over to the double leading scheme has very little effect.

b) Consequently, the data cannot yet fix the value of x_0, however if x_0 is as large as 0.1 they favour Q_0 factorization over the standard one.

c) In general both the relative and absolute sizes of λ_q, λ_g depend strongly on expansion, fitting, renormalization and factorization schemes. In particular if $x_0 = 0$ in $\overline{\mathrm{MS}}$ fitting $\lambda_q \simeq \lambda_g$ (within errors), but in DIS fitting $\lambda_q > \lambda_g$; while if $x_0 = 0.1$ in $\overline{\mathrm{MS}}$ fitting $\lambda_q > \lambda_g$, but in DIS fitting $\lambda_q \simeq \lambda_g$.[¶] The exception is that λ_q when fitted in DIS is independent of expansion, renormalization and factorization scheme, since it is directly related to the small-x behaviour of a physical observable, F_2.

d) The scheme dependence of λ_g is least severe in Q_0 factorization, where the gluon is generally rather flat. This provides phenomenological support to the theoretical expectation[10] that the input to perturbative evolution is of more direct physical significance in this scheme.[‖]

e) Whereas at fixed starting scale schemes with larger x_0 tend to have somewhat smaller values of λ (i.e. less singular inputs) the main effect of going over to the double leading scheme is to reduce the sensitivity to the starting distribution: double scaling then results ¿from a rather wide range of boundary conditions.

f) Conversely, there is a very large scheme dependence at large ρ (i.e. close to the boundary of perturbative evolution) which may signal a breakdown of leading-twist perturbative calculations there. This makes a perturbative

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[§]The starting scale should also be treated as a free parameter; it turns out however that a good fit can be obtained within quite a wide range of values of Q_0, the resulting values of λ being decreasing functions of Q_0.

[¶]This seems to disagree with ref. [12] where (on the basis of an $\overline{\mathrm{MS}}$ calculation at two loops) it is claimed that λ_g is significantly smaller than λ_q: it also suggests that some of the assumptions made in the discussion of the relative size of λ_q and λ_g in ref.[11] are incorrect.

[‖]The "initial Pomeron" reconstructed from the best-fit initial gluon distribution according to ref.[10] appears then to be soft.

reconstruction of the input parton distribution (and in particular the input gluon) from a measurement of the evolved structure function very difficult. Which is as it should be: evolving to smaller x and/or lower Q^2 leads one eventually into the intrinsically nonperturbative region.
g) Direct measurements of F_2 at larger values of ρ may help to reduce this ambiguity (or at least postpone it to yet larger values of ρ) by putting constraints on x_0. However, if the new data deviate strongly from the two loop curve this might suggest a breakdown of leading twist perturbation theory in this region.

Finally we note that when the physical parameter α_s is also included in the fit, its value turns out to be largely insensitive to all of these scheme ambiguities, thereby allowing a determination of it from small-x structure function data alone[8].

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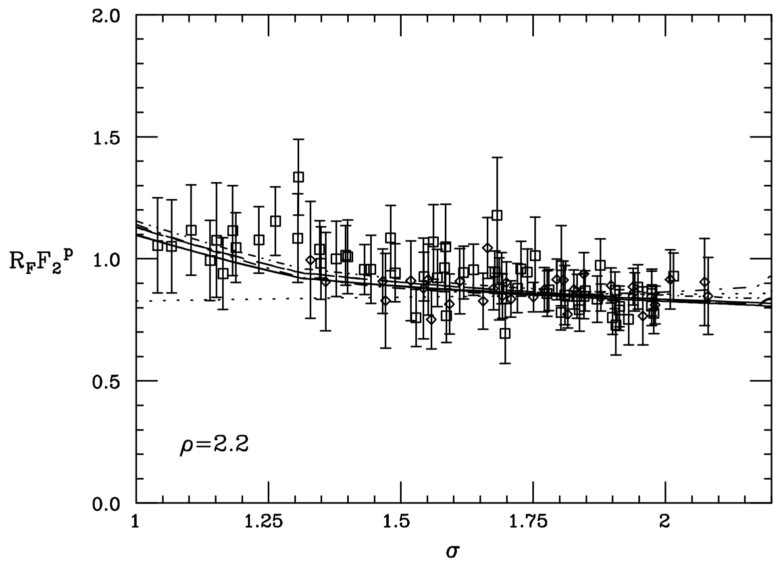

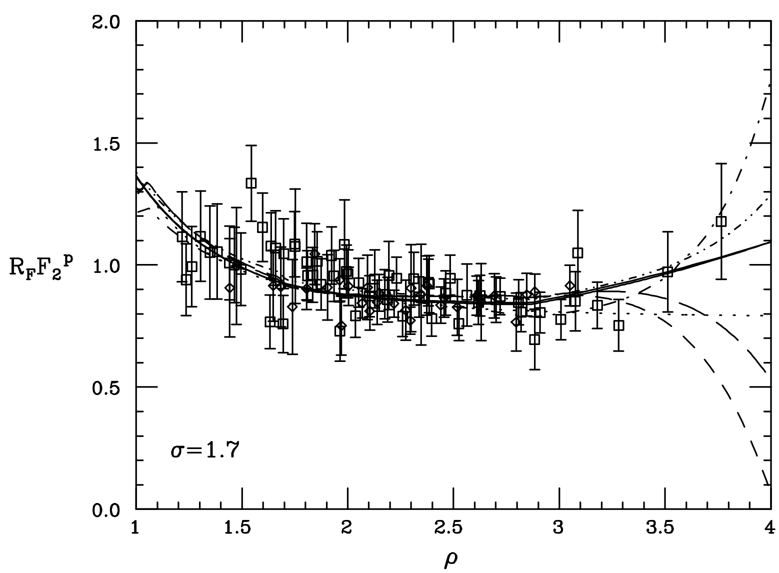

Figure 1: Scaling plots corresponding to double scaling (dotted); two loops (double leading with $x_0 = 0$) $\overline{\mathrm{MS}}$ or DIS (solid); double leading $\overline{\mathrm{MS}}$ (dot dash), $Q_0\overline{\mathrm{MS}}$ (double-dot dash), Q_0DIS (long dashes), DIS (short dashes). The double scaling curve is from ref.[4], the other curves correspond to entries denoted by * in the table.